MULTIPLE REGION FDTD (MR/FDTD) AND ITS APPLICATION TO MICROWAVE ANALYSIS AND MODELING

J. Michael Johnson
johnson@ee.ucla.edu

Yahya Rahmat-Samii
rahmat@ee.ucla.edu

Department of Electrical Engineering
University of California, Los Angeles
Los Angeles, CA 90095-1594

ABSTRACT

Multiple Region FDTD (MR/FDTD), an extension of classical FDTD to multiple sub-regions within a problem domain, is introduced. In MR/FDTD the problem domain is broken into several independent FDTD sub-region lattices. The sub-region lattices are terminated using a single surface integral radiation boundary condition applied simultaneously to all sub-regions providing mutual interaction between the sub-regions. The advantages of MR/FDTD for sparse modeling problems include computational and memory efficiencies that result from confining the FDTD lattices to the space near objects and the ability to use different lattices and/or lattice orientations within each sub-region. MR/FDTD also eliminates the need for local absorbing boundary conditions.

INTRODUCTION

Finite Difference Time Domain (FDTD), first introduced by Yee [1] and later extended and improved by others [2-5], is a powerful, robust, and popular modeling algorithm based on the direct numerical solution of Maxwell's Equations in the differential, time domain form. While very effective, FDTD can suffer from computational and memory usage inefficiencies when the spacing between objects in the computational domain becomes large yielding a sparse modeling problem. In addition, modeling inefficiencies and/or errors can result when object boundaries are not well matched to the FDTD lattice.

These inefficiencies and errors in classical FDTD result from the use of a uniform, space filling, typically orthonormal, lattice applied throughout a generally convex shaped problem domain. In a typical FDTD modeling problem a great deal of calculation time and computer memory is devoted to determining the fields at free space lattice points between objects to provide field continuity. The actual fields at most of these points are, in general, of little or no interest to the problem at hand. The inefficiencies of FDTD are often exacerbated by the boundary conditions used to terminate the lattice. These boundary conditions, such as the Mur absorbing boundary conditions (ABC) [4], normally require a buffer layer between the boundary and problem space region to achieve acceptable accuracy levels. The buffer layer adds a disproportionately large number of lattice points since it lies around the outer edges of the problem domain. Varying the lattice to better fit elements within the model domain is also a problem due to concerns about artificial reflection and dispersion errors introduced by variations in the lattice.

The Multiple Region FDTD (MR/FDTD) approach introduced in this paper seeks to avoid the inefficiencies of classical FDTD when applied to large, sparsely filled, modeling problems by eliminating explicit calculation of fields in the space in-between sub-regions surrounding the modeled objects. In MR/FDTD the problem space is divided into several, independent sub-regions distributed in otherwise free space. The fields in the sub-regions are determined using localized FDTD lattices and the interaction between the sub-regions is accounted for by performing a surface integration of equivalent sources on the sub-region surfaces. These equivalent sources are determined using the Shelkunoff equivalence and their integration yields an exact terminating boundary condition, thereby eliminating the need for artificial absorbing boundary conditions.

THE TIME DOMAIN SURFACE EQUIVALENCE PRINCIPLE

The Schelkunoff surface equivalence theorem states that the real electromagnetic sources and/or scatterers inside an imaginary volume can be replaced by equivalent magnetic and electric current sources on the surface enclosing the volume resulting in the same fields outside the volume and a zero field inside the volume.

0-7803-3246-6/96/$5.00 © IEEE

1996 IEEE MTT-S Digest

The presence of a zero field inside this exclusion volume, in turn, allows its contents (sources and scatterers) to be replaced by free space. With the entire problem domain now filled by free space, the fields anywhere outside the imaginary volume can be found by integrating the equivalent sources over the surface using the free space Green's function.

Equation (1) gives the electric field everywhere in the free space region outside the exclusion volume in terms of equivalent surface currents on the bounding surface. The surface current densities $\boldsymbol{M}_s$ and $\boldsymbol{J}_s$ are related to the tangential components of the fields on the surface by $\boldsymbol{M}_s(\mathbf{r}',t) = -\hat{\mathbf{n}} \times \mathbf{E}(\mathbf{r}',t)$ and $\boldsymbol{J}_s(r',t) = \hat{\mathbf{n}} \times \mathbf{H}(\mathbf{r}',t)$.

$$\begin{aligned} \boldsymbol{E}(\mathbf{r},t) = \frac{-1}{4\pi\varepsilon}\int_s \Bigg\{ & \left(\left[\frac{1}{c^2 R}\frac{\partial}{\partial\tau}\boldsymbol{J}_s(\mathbf{r}',\tau) + \frac{1}{cR^2}\boldsymbol{J}_s(\mathbf{r}',\tau)\right]\cdot\frac{\boldsymbol{R}}{R}\right)\frac{\boldsymbol{R}}{R} \\ & + \mu\varepsilon\left(\frac{1}{R}\frac{\partial}{\partial\tau}\boldsymbol{J}_s(\mathbf{r}',\tau)\right) \\ & + \varepsilon\left(\frac{1}{R^2}\boldsymbol{M}_s(\mathbf{r}',\tau) + \frac{1}{cR}\frac{\partial}{\partial\tau}\boldsymbol{M}_s(\mathbf{r}',\tau)\right)\times\frac{\boldsymbol{R}}{R}\Bigg\} ds' \end{aligned} \quad (1)$$

where:

$$\tau = t - \frac{R}{c} \qquad \boldsymbol{R} = \mathbf{r} - \mathbf{r}'$$

$$R = |\mathbf{r} - \mathbf{r}'| \qquad c \equiv \frac{1}{\sqrt{\varepsilon\mu}}$$

An equation for $\mathbf{H}(\mathbf{r},t)$ can be found from equation (1) by using duality. The unit vector $\hat{\mathbf{n}}$ is normal to the surface at $\mathbf{r}'$ and pointing out of the exclusion volume.

The equivalence principle is readily extended to multiple, simply connected, exclusion volumes, $\mathbf{V}_1, \mathbf{V}_2, \ldots \mathbf{V}_n$. As with one volume, the determination of the fields outside the volumes, $\mathbf{V}_1, \mathbf{V}_2, \ldots \mathbf{V}_n$, amounts to the integration of the tangential components of the fields and their time derivatives over all of the enclosing surfaces, $\mathbf{S}_1, \mathbf{S}_2, \ldots, \mathbf{S}_n$.

Alternate equations to (1) for the fields outside the exclusion volumes that may be useful in implementing MR/FDTD can be formulated in several different but essentially equivalent ways. One such alternate formulation for $\mathbf{E}(\mathbf{r},t)$ applicable to traditional Cartesian coordinate system based FDTD is the Kirchhoff Integral formula (2).

$$\boldsymbol{E}(\mathbf{r},t) = \frac{1}{4\pi}\int_s \left\{ (\hat{n}\cdot\hat{R})\left[\frac{\boldsymbol{E}(\mathbf{r}',\tau)}{R^2} + \frac{1}{cR}\frac{\partial \boldsymbol{E}(\mathbf{r}',\tau)}{\partial\tau}\right] - \frac{1}{R}\frac{\partial \boldsymbol{E}(\mathbf{r}',\tau)}{\partial n} \right\} ds' \quad (2)$$

The use and implementation of the Kirchhoff Integral formula (2) as an integrated radiation boundary condition (IRBC) is presented in [6]. This formulation requires only knowledge of the $\mathbf{E}(\mathbf{r}',t)$ however a derivative in space as well as in time is required and past values of the both the field and its space derivative must be saved to enable evaluation of the retard time field that are a function of τ. When used in a single region IRBC case described in [6], high levels of accuracy and reasonable computational and memory efficiencies are achieved. Yet another formulation [7] that replaces the time derivative with a time integration has been proposed and provides similar accuracy improvements over the Mur ABC. Elimination of the time derivative comes at the expense of requiring a second order derivative in the space domain.

MULTIPLE REGION FINITE DIFFERENCE TIME DOMAIN

A typical FDTD modeling problem that includes a number of scatterers with a source is the illustrated in Figure 1. To use classical FDTD to model this problem, an FDTD lattice must be established with fine enough spacing to resolve not only the locations of the modeled elements but also to resolve important features of the source and scatterers including slanted edges, notches and/or holes. In addition, classical FDTD requires that a single, uniform grid be established that completely fills the region. Finally a buffer layer must be added to the outside edge of the model space prior to the application of an absorbing boundary condition to terminate the lattice. Consequently, a great deal of computational time and memory is devoted to a large number of free space grid points.

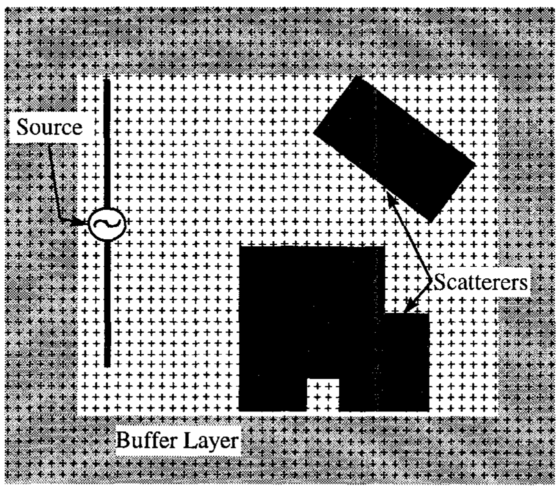

Figure 1. A space filling, often an excessively fine grid is necessary in classical FDTD to adequately represent modeled objects.

Now consider the same problem analyzed using MR/FDTD as illustrated in Figure 2. In the MR/FDTD method, the FDTD problem domain is broken into a number of interior problems consisting of solving for fields in independent sub-regions using FDTD and an exterior problem in which the fields are found outside the sub-regions by performing a surface integration of equivalent currents. Classical FDTD is applied to each of the sub-regions with the boundary layer calculation utilizing either equation (1), equation (2) or one of the several other alternate forms.

In MR/FDTD lattice spacing and even coordinate systems within sub-regions may be chosen to best suit the modeling requirements of that region. Time stepping fields are calculated only within the sub-regions of

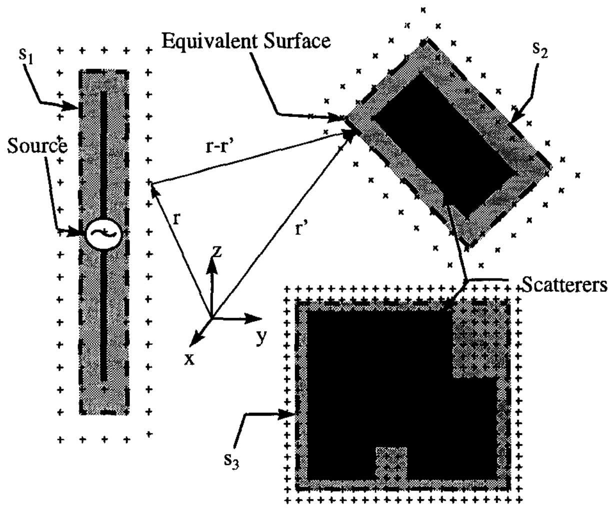

Figure 2. The fields on all sub-region bounding surfaces, S1, S2, and S3, are integrated at each time step to determine the new electric fields on the boundary layer lattice points for the next time step.

interest and not in the spaces between the sub-regions. Use of equation (1) terminates the FDTD lattice of the sub-regions enclosed in surfaces, S_1, S_2, ... , S_n, providing an exact radiating boundary condition eliminating the need for buffer layers. Lattices are established only in close proximity to the actual objects being modeled in each of these sub-regions. These factors greatly reduce the total FDTD lattice area in large sparse problem domains and allow more flexibility in lattice application.

The example depicted in Figure 2 has 3 sub-regions (shaded areas) surrounded by the surfaces $\mathbf{S}_1$, $\mathbf{S}_2$, and $\mathbf{S}_3$. The independent sub-regions in Figure 2 have different lattices adapted to the objects they contain. The lattice spacing in sub-region 3 is smaller than sub-regions 1 and 2 to accommodate the notches in the modeled scattering object. The grid of sub-region 2 is rotated to better align with the object that it contains. If need be, lattices based on different orthogonal, curvilinear coordinate systems may be applied as required in given sub-regions. No lattice is established in the free space region between sub-regions.

Once the grids are established, the MR/FDTD algorithm proceeds in a manner that is very similar to the classical FDTD. At each time step, the fields at all grid points are updated using past field values. If the point in question lies entirely inside a sub-region or on a sub-region surface, the classical FDTD update equations are used. If the point is a boundary layer or lattice termination point either equation (1) or its dual is used to calculate the updated field values. Near field values at points other than the boundary points can also be found using equations (1) and/or its dual and far field radiation patterns can be obtained in a similar manner using the field values available for the equivalent surfaces.

Both the $\mathbf{E}(\mathbf{r}',t)$ and the $\mathbf{H}(\mathbf{r}',t)$ on the equivalent surfaces are required to find the new fields using equation (1). If the surfaces are chosen to pass through a plane containing the tangential $\mathbf{E}(\mathbf{r}',t)$ field components for example, the tangential $\mathbf{H}(\mathbf{r}',t)$ fields can be found by interpolating between values on the two adjacent $\mathbf{H}$ field planes. A linear interpolation provides second order accuracy (error is $O(\Delta^2)$) that is consistent with the second order accuracy of the center difference derivatives normally used in FDTD.

Since the fields required by (1) are time delayed, previous values of the fields on the surfaces from past time samples must be retained. The number of past time samples is determined by the largest $\mathbf{R}$ ($\tau_{max} = t - R_{max}/c$) in a given problem. In addition, since FDTD utilizes a time stepping approximation to linear time ($t=n\Delta t$), both the field values and their time derivatives must be interpolated from available values. While a linear interpolation is adequate for the time values themselves, a second order or three point interpolation must be used to maintain second order accuracy in the time derivatives.

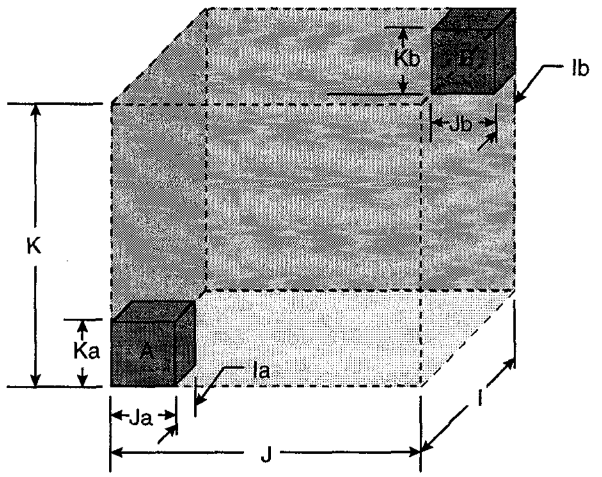

Figure 3: Example problem for comparing MR/FDTD and classical FDTD consisting of two sub-regions, A and B, located in an overall region.

A simple comparison of the memory required by classical FDTD and MR/FDTD reveals that significant savings can be achieved in some classes of problems by using MR/FDTD. Consider a typical problem depicted in Figure 3 where the sources and scatterers are confined to two sub-regions, A and B. Figure 4 is a plot memory efficiency of MR/FDTD relative to classical FDTD where the space between the sub-regions is varied. In the illustrated case, memory efficiency is defined as the ratio of memory required by MRFDTD to that required by FDTD. As can be seen in Figure 4, MR/FDTD can reduce the memory required by a factor of 2 to 5 for many problems.

$$C(s,Ia)=\frac{MRFDTD\_Bytes(s,Ia)}{FDTD\_Bytes(s,Ia)} \quad Ib=Jb=Kb=Ja=Ka=Ia \quad I=J=K=s\cdot Ia$$

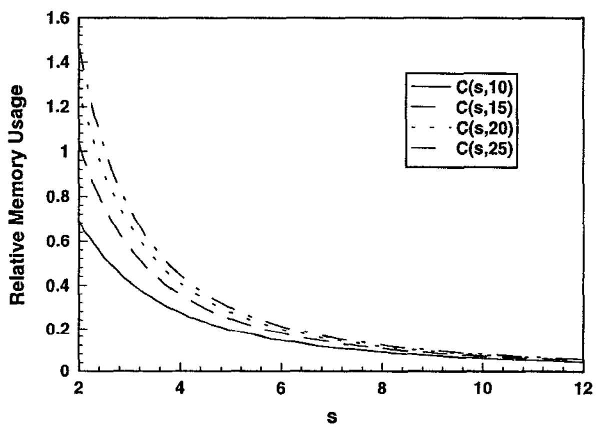

Figure 4: Example problem for comparing MR/FDTD and classical FDTD showing that for some dimensions MR/FDTD can save significant memory.

CONCLUSION

This paper has introduced the concept of Multiple Region FDTD and discussed some of the details of its development and implementation. MR/FDTD has significant advantages over classical FDTD for some classes of problems since lattices, possibly with different orientations and are applied independently to the sub-regions and since fields away from the regions of interest are not calculated. Comparisons between MR/FDTD and classical FDTD have been presented for a typical problem in terms of memory usage.

Acknowledgments– This work was supported in part by ARPA Contract # JFBI94-222/J4C942220.

REFERENCES

[1] Yee, K. S., "Numerical Solution of Initial Boundary Value Problems Involving Maxwell's Equations in Isotropic Media," *IEEE Trans Antennas Propagat.*, vol. AP-14, pp. 302-307, May 1966.

[2] Taflove, A and M. E. Bodwin, "Numerical Solution of Steady-state Electromagnetic Scattering Problems Using the Time-dependent Maxwell's Equations," *IEEE Trans. Microwave Theory Tech.*, vol. 23, pp. 623-630, 1975.

[3] Kunz, K. S. And K. M. Lee, "A Three-Dimensional Finite-difference Solution of the External Response of an Aircraft to a Complex Transient EM Environment," I. The method of its implementation. II. Comparisons of predictions and measurements, *IEEE Trans. Electromag. Compat.,* 20, pp. 238-333, 1975.

[4] Mur, G. "Absorbing Boundary Conditions for Finite Difference Approximations of the Time-Domain Electromagnetic Field Equations," *IEEE Trans. Electromag. Compat.* EMC-23, pp. 1073-1077, Nov. 1981.

[5] Jensen, M. A. and Y. Rahmat-Samii, "Performance Analysis of Antennas for Hand-Held Transceivers using FDTD," *IEEE Trans. Antennas Propagat.*, vol. 42, no. 8, pp. 1106-1113, August 1994.

[6] De Moerloose, J. and D. De Zutter, "Surface Integral Representation Radiation Boundary Condition for the FDTD Method," *IEEE Trans. Antennas Propagat.*, vol. 41, no 7, pp. 890-896, July 1993.

[7] Olivier, J. C. "On the Synthesis of Exact Free Space Absorbing Boundary Conditions for the Finite-Difference Time-Domain Method," *IEEE Trans. Antennas Propagat.*, vol. 40 no. 4, pp. 456-460, April 1992